Piercing the Cloud of Unknowing:
The Discrete Revolution of a Discreet Revolutionary

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Abstract

On the threshold of the 20th century, the broad consensus among physicists was that the body of
knowledge comprising their branch of science was ostensibly complete. There appeared to be no question
concerning the material universe for which physics could not provide an answer. It came as a profound
shock, therefore, when some phenomena were observed which could not be explained by classical physics.
Nevertheless, it was expected that these ‘clouds’, which obscured the beauty and clarity of physics, would
ultimately yield to solution given the time and patience needed to unravel the complexity of real physical
situations. Such optimism, however, proved unfounded.

Within a few years, the foundations of physics had been undermined and the revolution that ensued
marked a distinct break with the classical physics that preceded it. In this paper, we examine one of these
‘clouds’ and attempt to answer a number of questions. What was the nature of this ‘cloud’? What was
the reaction of scientists at the time, and how did they attempt to dispel the ‘cloud’? How did a reluctant
revolutionary propose a speculative solution? What were the implications of this proposal, and why did it
have such far-reaching consequences for science and philosophy?

Keywords: Blackbody radiation; Max Planck; Quantum mechanics; Ultraviolet catastrophe

“An important scientific innovation rarely makes its way by gradually winning
over and converting its opponents: it rarely happens that Saul becomes Paul.
What does happen is that its opponents gradually die out, and that the growing
generation is familiarised with the ideas from the beginning.”

1. Introduction

A few years before the outbreak of the First World War, a wealthy Belgian industrialist and philanthropist,
Ernest Solvay (1838-1922), sponsored the first in a series of international conferences on physics. Attendance
at these meetings in Brussels was by special invitation only, and the participants – usually limited to thirty –
were asked to concentrate on a pre-arranged topic. In bringing together the world’s most renowned physicists,
the conferences “served as sites for powerful reviews of the field” and were “catalysts for intellectual and
social networks” (Galison 2007). The first five meetings, all of which were chaired by the distinguished
Dutch physicist Hendrik Lorentz (1853-1928), between 1911 and 1927, chronicled in a most remarkable way
the development of 20th century physics (see Marage and Wallenborn 1999; Berends 2015; Coupain 2015).

The theme of the first congress, in autumn 1911, was “the theory of radiation and the quanta”, at a time

when the foundations of physics were totally shaken (Straumann 2011). The 23 attendees included Max Planck (1858-1947) and Albert Einstein (1879-1955), both of whom were initiators of this disruption (Lambert 2015), although they were different temperamentally and seemed to exemplify a distinction in human nature made famous by Isaiah Berlin (1953) in the epigram: “the fox knows many things, but the hedgehog knows one big thing.” At the start of the 20th century, Planck was the most famous physicist in Germany. Conservative, modest and retiring by nature, in his research Planck seemed to personify the hedgehog, whose purpose is to know one big thing and to strive without ceasing to give reality a unifying shape: “who relate everything to a single central vision, one system, less or more coherent or articulate, in terms of which they understand, think and feel – a single, universal, organising principle in terms of which alone all that they are and say has significance” (Berlin 1953: 2).

In contrast, Einstein, the second-youngest participant of the 1911 Solvay meeting, had achieved recognition at the age of 26 with the publication of four ground-breaking papers, each on a different subject, and went on to receive worldwide acclaim after confirmation of his general theory of relativity by the Eddington experiment of 1919. In his work, Einstein appeared to characterise the fox, who pursues many ends, often unrelated and even contradictory, but who comes to understand that a coherent worldview is probably beyond him - as Einstein found in his fervent pursuit of a unified field theory that, he hoped, would supplant quantum mechanics (see Pais 1982; Home and Whitaker 2007; Lindley 2007; Newton 2009).

The fifth Solvay congress of 1927, on the theme of “electrons and photons”, is regarded as the most illustrious assembly of scientists in history. It was devoted to quantum theory, and was attended by no less than nine theoretical physicists who had made fundamental contributions to the theory, including Planck and Einstein. Each of the nine would eventually be awarded a Nobel Prize for his contribution; and, in fact, of the 29 attendees at the 1927 conference, 17 were, or would ultimately become, Nobel Prize laureates. These pioneering scientists proved the value of quantum theory by explaining a wide range of phenomena, to such an extent that it is recognised as “the most successful scientific theory ever produced” (Davies and Brown 1986: 1). Considering the time scale of the early Solvay convocations (interrupted as they were by the cataclysm of the Great War), it is no exaggeration to say that there is hardly any other period in the history of science in which so much was clarified by so few in so short a time.

2. Two Clouds

On the threshold of the 20th century, the broad consensus among physicists was that the body of knowledge comprising their branch of science was ostensibly complete. Since the start of the scientific revolution in the 16th century, an impressive collection of learning - comprising instruments, experiments, observations, techniques, laws and theories - had been accumulated, analysed and classified into distinct fields: kinematics, dynamics, mechanics, thermodynamics, electricity, magnetism, acoustics and optics. In particular, the towering edifices of Newton’s laws of motion and universal gravitation in the 17th century (culminating with the Principia in 1687), and Maxwell’s laws of electromagnetism in the 19th century, had been scrutinized, confirmed, and successfully accounted for natural phenomena. The primary objects of study, matter (particles) and energy (waves), were thought to be distinct and separate phenomena. All

2 “The fox knows many things; the hedgehog knows one big thing” is a fragment of verse by the 7th-century BC Greek poet Archilochus, which became the animating principle for the celebrated essay by Berlin (1953).
3 On Planck’s nature and his approach to research, see Heilbron (1986).
4 The Eddington experiment - named after the British astronomer Arthur Stanley Eddington - was an observational test of general relativity conducted in 1919. The aim was to measure the gravitational deflection of starlight passing near the Sun. Einstein had predicted the value of this deflection, and it was one of the tests proposed for his 1915 theory of general relativity.
that remained was to tidy up some loose ends; physicists were by-and-large engaged in routine work, or the 'normal science' identified by Kuhn (1962: 10) and characterised as: “research firmly based upon one or more past scientific achievements” and which entailed puzzle-solving (Kuhn 1962: 35-42). Any promising student considering a career in physics for a sense of awe, novelty and discovery might well have been advised to pursue other avenues: “In this field, almost everything is already discovered, and all that remains is to fill a few important holes.”

According to Albert Michelson (1852-1931), the first American to receive a Nobel Prize in science (physics, 1907), speaking in 1894:

> It seems probable that most of the grand underlying principles have been firmly established … the future truths of physical science are to be looked for in the sixth place of decimals.⁵

Similarly, Lord Kelvin (1824-1907), speaking at the Royal Institution on 27 April 1900, is reported to have declared:

> There is nothing new to be discovered in physics now. All that remains is more and more precise measurement. … The beauty and clearness of the dynamical theory of heat and light – [of thermodynamics and electromagnetism] - is at present obscured by two clouds.⁶

But the fin de siècle saw a series of remarkable milestones in physics: the serendipitous discovery of X-rays by Röntgen in 1895; the chance detection of radioactivity by Becquerel in 1896; the recognition and identification of the electron by J. J. Thomson in 1897; and the discovery of radium by Marie and Pierre Curie in 1898. Hence, the prevailing wisdom had been wrong; as Robert Millikan remarked: “We had not come quite as near sounding the depths of the universe, even in the matter of fundamental physical principles, as we thought we had.” (Millikan 1951: 11).

Nevertheless, the “two clouds” to which Kelvin referred remained stubbornly opaque and impervious to resolution. One of these clouds was the inability of physicists to detect the luminiferous ether, “an ineffable something that pervaded everything - even the spaces between atoms” (Johnson 2008: 112). The ether was postulated by classical theory as the medium for the propagation of light and all radiation through space. This cloud appeared particularly impenetrable following the negative outcome of the celebrated Michelson-Morley experiment of 1887, dubbed “the most famous failed experiment in history” (Blum and Lototsky 2006: 98). The enigma of the missing ether was only resolved with the publication of Einstein’s 1905 paper, ‘On the Electrodynamics of Moving Bodies’, in which he postulated that the speed of light is observed to be the same for any inertial frame, and that no ether frame is necessary. This gave us one of the landmark achievements of 20th century physics: Einstein’s theory of relativity. Michelson subsequently viewed the

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⁵ Philipp von Jolly, advising his student, Max Planck, not to pursue physics at the University of Munich. In 1878, Planck, as a graduate student, had come across a collection of papers by Rudolf Clausius and was so entranced that he commenced a thesis that critiqued the existing formulations of the second law of thermodynamics. By then, thermodynamics was regarded as virtually complete, and not an exciting or promising field for a young scientist.

⁶ Convocation Address at the Dedication of the Ryerson Physical Laboratory at the University of Chicago, ‘Some of the Objects and Methods of Physical Science’ (4 Jul 1894), published in University of Chicago Quarterly Calendar (August 1894), 3, No.2, 15.


⁸ Einstein’s paper was published as ‘Zur Elektrodynamik bewegter Körper” in the journal Annalen der Physik, 17: 891, 1905.
ether as “one of the grandest generalizations of modern science – of which we are tempted to say that it ought to be true even if it is not” (Michelson 1961: 162). Despite the result of their experiment, Michelson and Morley “had proved, contrary to their expectations, that there is no fixed backdrop of space, or even of time.” (Johnson 2008: 120).

The second cloud obscuring the “beauty and clearness” of physics was the blackbody radiation problem: the conventional and prosaic question of how objects radiate heat. This was broached in 1859, the same year that Darwin published his *Origin of Species*. Gustav Kirchhoff (1824-1887), a German experimental physicist, declared that determining the energy spectrum of a blackbody was the holy grail of physics. This conundrum concerned the thermodynamics of the exchange of energy between radiation and matter; specifically, an anomaly in the distribution of radiation emitted when a body is heated (Matthews 1978). When heated, bodies emit light at progressively higher frequencies. In equilibrium, the light emitted ranges over the whole spectrum of frequencies, with a spectral distribution that depends both on the frequency (or, equivalently, on the wavelength) of the light, and on the temperature. The spread of frequencies radiated at a particular temperature has a similar form, known as blackbody radiation. Most light energy radiates around one peak frequency, which scales with temperature from red towards blue. The result is an asymmetric ‘hill’-shaped spectrum, known as the blackbody curve.

By the late 19th century, physicists were well aware of blackbody radiation and had measured its frequency pattern, but they could not explain it. Two separate, contradictory theorems in classical physics attempted to describe this distribution: Wilhelm Wien (1864-1928) concocted an equation in 1896 that accounted for experimental results in the region of short-wavelength (high-frequency; blue spectrum) radiation; and Lord Rayleigh (John William Strutt, 1842-1919) and Sir James Jeans (1877-1946) proposed a law in 1900 that accounted for those in the region of long-wavelength (low-frequency; red spectrum) radiation. Neither of these formulae, however, matched the data for the distribution of power radiated by a blackbody at various temperatures across the whole range of frequencies (see Gasiorowicz 1974; Greiner 1989). This was cause for concern because both theories relied directly on the logical structure of classical physics. Rayleigh and Jeans’ proposed solution was particularly problematic, since their theory predicted an infinite release of energy at shorter wavelengths. This problem became known as the ‘ultraviolet catastrophe’.

The first fissure in the rigorous, reassuring edifice of classical physics, to which Michelson and Kelvin alluded, emerged in December 1900 when Planck delivered a scientific paper, on thermal radiation, to the *Deutsche Physikalische Gesellschaft* (German Physical Society) in Berlin (Gamow 1966; Matthews 1974). Conventional and orthodox by temperament, and with an abiding fascination for thermodynamics, Planck had been studying the blackbody radiation problem. While this was a matter of theoretical interest to late-19th century physicists, it scarcely seemed likely to require an entire new Weltanschauung. Indeed, Planck’s proposal excited no initial controversy or disquiet whatsoever; and its ramifications only became clear some years later when Einstein used Planck’s quantum hypothesis as a general aspect of microscopic reality in one of his landmark *annus mirabilis* papers of 1905.10

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10 Einstein published four seminal papers in the journal *Annalen der Physik* in 1905.
3. Entropy

During the final years of the 19th century, a number of physicists began to query the validity of the mechanical worldview, which until then had been taken for granted. The crux of the matter was whether classical, deterministic Newtonian mechanics could still be regarded as a valid description of all of nature (see Susskind and Hrabovsky 2013). Central to this inquiry, which probed the foundations of physics, was thermodynamics, and, in particular, the relationship between the laws of mechanics and the two basic laws of heat: the principle of energy conservation, and the second law of thermodynamics. In effect, two basic quantities, internal energy and entropy, are defined by the two laws of thermodynamics.

Planck was deeply interested in the second law of thermodynamics. His doctoral thesis from the University of Munich dealt with the second law, which was also the subject of most of his work until about 1905. According to the second law, in one of its several interpretations (Lynskey 2019), no process is possible in which the only result is the transfer of heat from a colder to a hotter body. Employing the concept of entropy, introduced on a purely phenomenological basis by Rudolf Clausius (1822-1888) in 1865, the law can be reformulated to state that the entropy of an isolated system always increases or remains constant – the notion of ‘irreversibility’ (Shankar 2014). The entropy $S$ is the state function whose differential is given by

$$
\frac{dS}{dT} \geq \frac{dQ}{T}
$$

where $dQ$ is the heat added to a system and $T$ is the absolute temperature. The equality holds for reversible processes and the inequality for irreversible ones.

Planck first encountered Clausius’s formulation in the second edition (1876) of his Mechanical Theory of Heat, and, thereafter, focused on entropy and how to understand irreversibility on the basis of the absolute validity of the second law.

One day, I happened to come across the treatises of Rudolf Clausius, whose lucid style and enlightening clarity of reasoning made an enormous impression on me, and I became deeply absorbed in his articles, with an ever increasing enthusiasm. … [Various setbacks] could not deter me from continuing my studies of entropy, which I regarded as next to energy the most important property of physical systems. Since its maximum value indicates a state of equilibrium, all the laws of physical and chemical equilibrium follow from a knowledge of entropy.13

Planck began his 1879 doctoral thesis, On the Second Law [Hauptsatz] of the Mechanical Theory of Heat, as follows:

While the first law of the mechanical theory of heat [thermodynamics], which expresses the equivalence of heat and work, is a consequence of the principle of the conservation of energy, and is therefore concerned with a quantity that remains constant in all the processes of the whole of nature, the second, by contrast, represents a law according to which nature is always constantly striving to carry out its processes always only in a certain sense – in a determinate direction – so that a return of the world to a

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11 Joule, Kelvin, Helmholtz and others first stated the principle of energy conservation in 1840-50; and the second law of thermodynamics was formulated by Clausius in 1850 and by Kelvin in 1853.
12 Planck’s doctoral thesis was entitled: Über den zweiten Hauptsatz der mechanischen Wärmetheorie (On the Second Principle of Mechanical Heat Theory) (1879).
previously occupied state is impossible. The significance of the second law, in its most general form, is
to fix mathematically the sense of this direction in which all the transformations of nature take place.¹⁴

In the 1890s, the debate about the second law of thermodynamics focused on the statistical (or probabilistic)
interpretation of matter that Ludwig Boltzmann (1844-1906) had originally proposed in 1872 and expanded
in 1877. This addressed the fundamental question of whether atoms and molecules were real and constituent
parts of matter, as Boltzmann’s theory presupposed (Cercignani 1998), or were simply heuristic tools for
solving problems. According to Boltzmann’s molecular-mechanical interpretation, the entropy of a system
is the collective result of molecular motions. It is a measure of the number of different arrangements of the
molecules (their position and momenta) in a given state of systems of the same kind with the same total
energy; entropy is thus a measure of probability, and hence the second law is valid only in a statistical sense.¹⁵

In contrast to what physicists working in thermodynamics had assumed all along, the second law,
fundamental though it remained, no longer laid down an irrefutable rule that the entropy of a closed system
could never decrease; it merely said that the probability for it to decrease is extremely low. For many
physicists this was a profound shock. The hypothesis that matter is made up of a huge number of minute
particles implied that some kinds of behavior of matter were not predictable with certainty, but only with high
probability (Jammer 1966; Newton 2009).

Planck’s belief in the absolute validity of the second law made him not only reject Boltzmann’s statistical
version of thermodynamics, but also doubt the atomic hypothesis upon which it rested. In the early 1880s,
he concluded that the atomic conception of matter was irreconcilably opposed to the law of entropy increase;
namely, processes in which the total entropy of a system decreased over time were to be considered strictly
impossible. Initially, he did not consider Boltzmann’s reformulation of the second law (into a statistical law)
as acceptable because Boltzmann’s statistical mechanics did not make the increase of entropy absolutely
certain, only highly probable. Planck’s opposition to atomism waned, however, during the late 1880s as he
realised the power of the hypothesis and the unification it brought to physical and chemical phenomena.¹⁶
Nevertheless, Planck’s attitude to atomism remained ambiguous, and he continued to give priority to
macroscopic thermodynamics and ignore Boltzmann’s statistical theory.

Boltzmann knew very well that my viewpoint was basically different from his. … The reason was that
at that time, I regarded the principle of the increase of entropy as no less immutably valid than the
principle of the conservation of energy itself, whereas Boltzmann treated the former merely as a law of
probabilities — in other words, as a principle that could admit of exceptions.¹⁷

By 1895, Planck was ready to embark on a major research project to determine thermodynamic
irreversibility without recourse to the atomic hypothesis. A utilitarian task at the time was to design the most
efficient light bulb possible, giving the most light while consuming the least electrical energy. Planck saw
that the key to this question was the fundamental issue raised by Kirchhoff in 1859 concerning the theoretical
construct known as a blackbody, which absorbs all electromagnetic radiation that falls on it. The question

¹⁴ Planck (1879): 1.
¹⁵ Boltzmann’s theory presupposed the existence of atoms and molecules, but this was vehemently challenged by several
influential advocates of ‘energetics’ – notably, Pierre Duhem (1861-1916), Georg Helm (1851-1923), Ernst Mach (1838-
1916) and Wilhelm Ostwald (1853-1932) - who wanted to free physics from the notion of atoms and base it on various
distinct types of energy.
¹⁶ This is evident, for example, in a three-part paper with the general heading “On the Principle of the Increase of Entropy”,
where Planck employed the entropy principle to solve problems of chemical affinity, spontaneity and equilibrium, and, in a
final installment, he applied the principle to a variety of electrochemical phenomena. See Planck (1887-91).
¹⁷ Planck (1949): 32.
was: how does such a body emit radiation? In particular, how does the intensity of the emitted radiation depend on its frequency and the body’s temperature?

This research project not only expressed Planck’s deep interest in the concept of entropy, but also displayed his ‘aristocratic’ attitude to physics: he often expressed the deep, esoteric conviction that the loftiest goal of any, if not all, science was the search for the absolute. This conviction was reflected in his focus on the fundamental aspects and disregard for more mundane, applied ideas.

I had always looked upon the search for the absolute as the noblest and most worthwhile task of science.\textsuperscript{18}

Planck believed that the principle of increasing entropy could be preserved intact as a rigorous theorem in a comprehensive worldview. When it came to the unexplained phenomena of blackbody radiation, it was this belief that acted as the criterion in his attempt at explanation. Planck’s fascination with entropy, which was shared by only a handful of other physicists, was not considered to be of central importance or of providing important results - until it did, with significant consequences.

My attention, therefore, was soon claimed by quite another problem, which was to dominate me and urge me on to a great many different investigations for a long time to come. The measurements made by O. Lummer and E. Pringsheim in the German Physico-Technical Institute, in connection with the study of the thermal spectrum, directed my attention to Kirchhoff’s Law, which says that in an evacuated cavity, bounded by totally reflecting walls, and containing any arbitrary number of emitting and absorbing bodies, in time a state will be reached where all bodies have the same temperature, and the radiation, in all its properties including its spectral energy distribution, depends not on the nature of the bodies, but solely and exclusively on the temperature. Thus, this so-called Normal Spectral Energy Distribution represents something absolute, and since I had always regarded the search for the absolute as the loftiest goal of all scientific activity, I eagerly set to work.\textsuperscript{19}

4. Thermal Radiation

There are two basic ways in which heat propagates in a given medium: conduction and radiation. Conduction is governed by a diffusion equation and the rate of change of temperature is proportional to the temperature gradient

\[
\frac{dT}{dt} = -\alpha \frac{d^2}{dx^2} T
\]

where \(\alpha\) is a constant that depends only on the properties of the medium, i.e., it does not depend on the temperature. Once the temperature is the same everywhere, the right hand side of this equation vanishes and there is then no conduction.

But in the absence of a surrounding fluid or of any solid connections through which heat could be conducted, a body can still lose heat energy. An isolated body in a vacuum cannot lose heat by conduction or convection, but it may lose heat by radiation, which is independent of any temperature gradient. This phenomenon is electromagnetic in origin. The energy that we receive from the Sun and most of the heat that one feels from a fire are examples of the transfer of heat by radiation.

Such thermal radiation is electromagnetic radiation emitted by a body solely on account of its temperature.

\textsuperscript{18} Planck (1949): 46.
\textsuperscript{19} Planck (1949): 34-35.
The radiation spans a continuous range of wavelengths, and the distribution of energy among these wavelengths depends on the temperature of the emitter. At temperatures below about 700K, the energy is associated almost entirely with infrared wavelengths; at higher temperatures, visible and ultraviolet wavelengths are also involved. Thermal radiation has all the general properties of electromagnetic waves: it can be reflected; its velocity in a vacuum is $3 \times 10^8 \text{ms}^{-1}$; it cannot be deflected by electric and magnetic fields; and the intensity of the radiation produced by a point source falls off as the inverse square of its distance from the source.

The rate of radiant heat loss from a body increases very rapidly with increasing temperature of the body. This phenomenon was studied experimentally by Josef Stefan (1835-1893), who compiled data from several sources. In a paper published in 1879, based on earlier observations by John Tyndall (1820-1893), Stefan showed empirically that the rate of heat loss per unit area by radiation from a body is proportional to the fourth power of its absolute temperature: the $T^4$ law.\(^\text{20}\) The same conclusion was reached by Boltzmann, Stefan’s former student, in 1884 from theoretical considerations. Thus, the power transferred per unit area of surface of a body of surface temperature $T$ by this process is given by the Stefan-Boltzmann law:

$$\frac{dP}{dA} \propto T^4$$

$$\frac{dP}{dA} = \varepsilon \sigma T^4$$

where $\varepsilon$ is a dimensionless number ($0 < \varepsilon < 1$) called the emissivity, which depends on the nature of the radiating surface, and $\sigma$ is a fundamental constant of nature called the Stefan-Boltzmann constant. The most efficient radiators, therefore, are surfaces for which $\varepsilon \to 1$ while highly polished surfaces are inefficient radiators and have low emissivity ($\varepsilon \to 0$). The value of the Stefan-Boltzmann constant is found from experiment to be

$$\sigma = 5.67 \times 10^{-8} \text{Wk}^{-4} \text{m}^{-2}$$

5. **Concept of a Blackbody**

In 1859, Kirchhoff demonstrated that the absorption of thermal radiation is found to be the exact reverse of the emission process. Highly polished surfaces ($\varepsilon \to 0$), for example, reflect the radiation rather than absorb it. But a surface that is a very efficient emitter ($\varepsilon \to 1$), on the other hand, will absorb radiation with equal efficiency. In the case of a surface for which $\varepsilon = 1$, all radiation which falls on it will be absorbed. Since such a surface absorbs light with the same efficiency as it does thermal radiation, it appears black, at least if it is not too hot, and hence is called a **blackbody** surface.

As the temperature of an object increases, it first gives out radiant heat without a change in colour - that is, it emits electromagnetic radiation in the infrared region of the spectrum. As the temperature increases further,
the object begins to glow red and then white, emitting visible electromagnetic radiation. All bodies at finite temperatures emit electromagnetic waves. Observation of the spectrum emitted by a solid shows that the radiation extends over a continuous range of frequencies, described as a *continuum*.

While the detailed form of the continuum emitted by a body depends to some extent on the composition of the body, Kirchhoff showed that, for bodies that absorb all thermal radiation that falls on them (blackbodies), the thermal emission spectrum depends only on temperature, i.e., the emitted intensity is a function of temperature $T$ and frequency $v$ only. Kirchhoff challenged experimental and theoretical physicists to discover the precise functional dependence on temperature and frequency. While the Stefan-Boltzmann law provided a partial answer, the search for an explanation of the frequency dependence at a given temperature engaged physicists for the subsequent half century, with revolutionary consequences.

A blackbody is a body that absorbs all the radiation that is incident on it, while reflecting none of it, so that any radiation emitted from the blackbody is a result of its overall temperature (Planck 1914: 10). The radiation of a blackbody corresponds to the condition of thermal equilibrium, and consequently to the maximum of entropy. Thermal equilibrium means the radiation is extremely uniform (the same at every point), isotropic (the same in all directions) and highly mixed, i.e., it contains many frequencies (Dunningham and Vedral 2011). The word ‘blackbody’ is actually misleading, because if this radiation peaks in the visible region of the spectrum, the body will appear to be the colour that corresponds to the peak frequency. This is, in fact, the case for the Sun, which to a good approximation is a blackbody whose spectrum peaks in the yellow part of the visible spectrum. By using the dependence of the blackbody spectrum on temperature, one can measure the temperature of an object by observing its colour.

The concept of a blackbody is, however, an idealised one. In practice, emitting solids are never perfect absorbers and, therefore, never perfect blackbodies. In principle, however, a heated, metallic oven (cavity) with matt black interior walls and a small aperture should approach the absorption properties required of a blackbody, because, after successive reflections within the cavity, any radiation entering the cavity is likely to be absorbed. The small amount of radiation that is reflected from the walls has very little chance of escaping through the aperture before it, too, is absorbed in a subsequent encounter with the wall. The emission spectrum of a cavity should, therefore, resemble closely that of an ideal blackbody.

One can observe and record the observed temperature dependence of blackbody radiation using such an arrangement. Measurements conducted on the radiation escaping from the small aperture in a heated cavity show that the intensity of the radiation varies strongly with the frequency. The dominant frequency shifts to a higher value (conversely, the dominant wavelength shifts to a lower value) as the temperature is increased. Inside a cavity, the radiation has nowhere to go and is continuously being absorbed and re-emitted by the walls. Thus, a small aperture will permit the escape of radiation emitted by the walls, not reflected, and therefore characteristic of a blackbody, i.e., a body that completely absorbs all the electromagnetic radiation falling on it.

When the cavity is at a low temperature, radiation is present (as infrared radiation) but invisible. At higher temperatures (say, 1000K), the frequencies reach the visible range and the cavity glows red. The maximum intensity of emitted blackbody radiation occurs at a frequency, $v_{\text{max}}$, which increases with temperature $T$. In 1893, Wien showed, on theoretical grounds, the following relationship (the Wien displacement law), a result confirmed by experiment.\(^{21}\)

\[
v_{\text{max}} \propto T
\]

\(^{21}\) Using this relationship between temperature and wavelength, and knowing that the predominant wavelength from the Sun is approximately 5nm, we find that the surface temperature, $T = 0.0029/5.0 \times 10^{-9} = 5800$K.
This law accounts for the changes in the colour of a solid with temperature. At low temperatures, \( v_{\text{max}} \) lies in the infrared region of the electromagnetic spectrum. As the temperature of the object increases, \( v_{\text{max}} \) moves into the red region of the spectrum – the solid glows red. Further increase in temperature causes the colour to change to orange, then yellow and blue and, finally, white, (i.e., all visible frequencies).²²

6. The Blackbody Radiation Problem

Following Kirchhoff’s challenge to the physics community in 1859, techniques to study the frequency distribution of the radiation emitted from a hot cavity gradually improved to the extent that it was possible to begin to distinguish different theoretical models. Perceiving that there was something absolute about blackbody radiation, various experimental and theoretical attempts were made in the 1890s to determine its spectral energy distribution - the curve displaying how much radiant energy is emitted at different frequencies for a given temperature of a blackbody. In particular, experimental results from several investigators in Berlin provoked certain theoretical developments.

In 1896, Planck’s colleague, Wilhelm Wien, and others at the Physikalisch-Technische Reichsanstalt (PTR, Imperial Bureau of Standards) in Berlin-Charlottenburg, conducted experiments to record the distribution of radiation emitted from the aperture of a blackbody (of porcelain and platinum), measuring from the near infrared into the violet. Elsewhere in the city, at Berlin’s Technische Hochschule, another of Planck’s close associates at the PTR, Heinrich Rubens (1865-1922), operated a different oven to measure into the deep infrared frequencies. The shapes of the radiation curves obtained were shown to be very similar to those calculated by James Clerk Maxwell (1831-1879) for the velocity (i.e., energy) distribution of heated gas molecules in a closed container. The question arose: could the blackbody radiation problem be understood in terms of Maxwell’s theory of electrodynamics, which was supposed to govern the behaviour of the microscopic oscillators that produced the heat radiation emitted by blackbodies? This envisaged electromagnetic waves (instead of gas molecules) bouncing around in equilibrium within the walls of a cavity.

I found a direct method for solving the problem in the application of Maxwell’s Electromagnetic Theory of Light. Namely, I assumed the cavity to be filled with simple linear oscillators or resonators, subject to small damping forces and having different periods; and I expected the exchange of energy caused by the reciprocal radiation of the oscillators to result, in time, in a stationary state of the normal energy distribution corresponding to Kirchhoff’s Law.²³

In 1896, Wien derived a formula, based on some speculative theoretical arguments, which agreed well with published experiments, but only in the high frequency part of the spectrum. The spectral density, i.e., the radiation energy density per unit frequency, was given by

\[
\lambda \propto \frac{1}{T}
\]

\[
T = \frac{0.0029}{\lambda}
\]

This was how early potters determined the temperature inside their kilns. Already in 1792, the famous porcelain maker Josiah Wedgewood had noted that all bodies become red at the same temperature. With increasing temperatures, the colour changed: 800K dark red; 1000K cherry red; 1200K orange; 1300K yellow; 1500K white.

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²³ Planck (1949): 35.
where $\nu$ is frequency, $T$ is temperature, and $A$ and $B$ are constants to be determined empirically. This formula is known as Wien’s distribution law.

Planck was particularly attracted to this formula, and he subsequently made a series of attempts to derive Wien’s distribution law on the basis of the second law of thermodynamics. By October 1900, however, his colleagues at the PTR - the experimentalists Otto Richard Lummer (1860-1925), Ernst Pringsheim (1859-1917), Ferdinand Kurlbaum (1857-1927) and Heinrich Rubens - had found definite indications that Wien’s law, while valid at high frequencies, broke down completely at low frequencies.

Planck learned about these results shortly before a meeting of the Deutsche Physikalische Gesellschaft on 19 October 1900. He knew how the entropy of the radiation had to depend mathematically upon its energy in the high-frequency region, if Wien’s distribution law held there. He also realised what this dependence had to be in the low-frequency region in order to reproduce the experimental results there. Planck deduced, therefore, that he should try to combine these two expressions, and transform the result into a formula relating the energy of the radiation to its frequency.

The result, which is known as Planck’s radiation law, was hailed as irrefutably correct. To Planck, however, it was simply a guess, a “lucky intuition” (Planck 1949: 41). In order to be taken seriously, it had to be derived from first principles. Planck applied himself immediately to this task, and by 14 December 1900, he succeeded—but at great professional sacrifice.

For this reason, on the very day when I formulated this law, I began to devote myself to the task of investing it with a true physical meaning. This quest automatically led me to study the interrelation of entropy and probability — in other words, to pursue the line of thought inaugurated by Boltzmann. Since the entropy $S$ is an additive magnitude but the probability $W$ is a multiplicative one, I simply postulated that $S = k \log W$, where $k$ is a universal constant; and I investigated whether the formula for $W$, which is obtained when $S$ is replaced by its value corresponding to the above radiation law, could be interpreted as a measure of probability.24

To achieve his goal, Planck found that he had to relinquish one of his most treasured beliefs: that the second law of thermodynamics was an absolute law of nature. Instead, he had to acknowledge Boltzmann’s interpretation, that the second law was a statistical law. In addition, Planck had to assume that the oscillators comprising the blackbody and re-emitting the radiant energy incident upon them could not absorb this energy continuously but only in discrete amounts, in quanta of energy; only by statistically distributing these quanta, each containing an amount of energy $h \nu$ proportional to its frequency, over all of the oscillators present in the blackbody, could Planck derive the formula he had concocted two months earlier in October 1900.

Now as for the magnitude $W$, I found that in order to interpret it as a probability, it was necessary to introduce a universal constant, which I called $h$. Since it had the dimension of action (energy x time), I gave it the name, elementary quantum of action. Thus the nature of entropy as a measure of probability, in the sense indicated by Boltzmann, was established in the domain of radiation, too.25

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24 Planck (1949): 41-42.
25 Planck (1949): 43.
7. The Ultraviolet Catastrophe

At the turn of the twentieth century, the English physicist Lord Rayleigh attempted to explain the observed blackbody spectrum classically by considering the behaviour of electromagnetic waves within a cavity (see Gasiorowicz 1974; Greiner 1989). It was known from electromagnetic theory that, because metallic surfaces are not able to support electromagnetic waves, the waves must have nodes at the walls of the cavity – they must be standing waves. In 1900, Rayleigh attempted to apply his earlier treatment of sound waves in a cubical enclosure to standing electromagnetic waves in a cavity. Rayleigh had shown that the number of acoustic modes with frequencies between $\nu$ and $\nu + d\nu$ was given by

$$N(\nu)d\nu = \frac{4\pi V}{v^2} d\nu$$

where $V$ is the volume of the cavity and $v$ is the wave velocity. In the case of electromagnetic waves, this expression needed to be modified to take into account the fact that there are two possible polarisation states and that, in a vacuum, $v = c$. Thus Rayleigh, with additional input from James Jeans, proposed that the distribution of frequencies in cavity radiation would be given by

$$N(\nu)d\nu = \frac{8\pi V}{c^3} \nu^2 d\nu$$

Rayleigh then assumed that the radiation in the cavity was in equilibrium with the oscillators in the walls that were the sources of the radiation. To determine the total energy of the radiation within the cavity, this equation must be multiplied by the average energy per standing wave frequency mode $\langle E \rangle$, which depends on the temperature $T$ of the cavity walls. The frequency distribution of the radiation energy density (energy per unit volume) is thus obtained by multiplying $N(\nu)d\nu$, given by the above equation, by $\langle E \rangle$ and dividing by $V$. Hence, the energy density of waves with frequency between $\nu$ and $\nu + d\nu$ is given by

$$\varrho(\nu, T)d\nu = \frac{8\pi}{c^3} \langle E \rangle \nu^2 d\nu$$

How to determine $\langle E \rangle$ and, in particular, how it depends on $T$, was a matter of some conjecture. Despite his own and others’ reservations about its applicability in this situation, Rayleigh applied the principle of equipartition of energy to the standing wave oscillations assuming two degrees of freedom, each contributing $\frac{1}{2} kT$. Thus, $\langle E \rangle = kT$ and the frequency distribution of energy density in the cavity at temperature $T$ is given by

$$\varrho(\nu, T)d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu$$

This is known as the Rayleigh-Jeans formula for cavity radiation.

The energy distribution of cavity radiation predicted by the Rayleigh-Jeans formula accounted for the observed blackbody spectrum at low frequencies (long wavelengths), but it failed completely to predict the observed fall-off at high frequencies (short wavelengths). In fact, the Rayleigh-Jeans equation predicted that the radiated energy would tend to infinity at high frequencies. Classical physics had, therefore, failed to account for the cavity radiation spectrum.
Why was this of significance? As mentioned previously, the properties of radiation from a cavity are of particular interest because they approach those of a blackbody. Thus, the failure of classical electromagnetic theory to account for cavity radiation was also expected to apply to blackbody radiation. The implications were considerable because, were the classical result to apply, any solid at a finite temperature would radiate infinite amounts of energy at high frequencies (in the ultraviolet region). If this prediction were correct, any solid would quickly cool to 0K through emission of electromagnetic radiation, and the universe, as we know it, would not be viable. On a more prosaic level, it cannot be true because every time we used an oven or a fire, we would be bombarded with ultraviolet radiation. A heated body would emit so much ultraviolet light so as to become invisible to the naked eye. Clearly, though, this does not happen: incandescent bodies are not invisible. This fundamental failure of classical theory to explain the decline in energy radiated at high frequencies in the ultraviolet region was described as the ‘ultraviolet catastrophe’.26

Clearly, since the theory did not align accurately with observations of reality from blackbodies, it needed to be discarded or revised. Classical electromagnetism, as powerful though it is, had some limitations in its ability to describe light and energy.

8. Planck’s Hypothesis

In order to explain the failure of classical physics to account for the observed emission spectrum of cavity radiation, Planck found it necessary to introduce a radically new hypothesis governing the behaviour of harmonic oscillators – the concept called quantization – that was completely at variance with classical physics.

In seeking an explanation of the observed frequency distribution of cavity radiation, he viewed the problem from a quite different perspective to that of Rayleigh. Planck concentrated more on the oscillators in the cavity walls, which he addressed through his own version of the probabilistic dynamical approach to the study of heat developed by Boltzmann in the 1870s. Underlying Planck’s treatment of the problem was the hypothesis that the oscillators, such as atoms in the walls of the cavity, can take only certain discrete energy values. These energies are given by Planck’s hypothesis, which states that a system which is performing simple harmonic motion of frequency $\nu$ can only have values of energy that satisfy the equation:

$$E_n = n\hbar \nu$$

where $n = 0, 1, 2, 3, \ldots$ (integers) and $\hbar$ is Planck’s constant ($6.63 \times 10^{-34}$ Js).

We know from classical physics that heat is the transfer of kinetic energy from one location to another. In the case of a solid hot metal, that kinetic energy takes the form of atomic vibrational motion or oscillations; it is these oscillations that take the form of the light we see in the blackbody spectrum. Planck proposed that the vibrational energies of the atoms, and by extension the energies of the electromagnetic waves emitted by these atoms, must be quantized, meaning that, instead of being able to take on any value from a continuous series, they can only possess specific, discrete values from a set of accepted values.

The $n$ value results in quantization because it can only be an integer, meaning that the resulting energies will also comprise a set of allowed values. In other words, all energies are multiples of the smallest fundamental unit of energy, the Planck energy.27

This application of quantization and the accompanying Planck’s constant were developed in ad hoc

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26 Paul Ehrenfest (1880-1933) coined the term ‘ultraviolet catastrophe’ in 1911.
27 Quantum theory went on to show that everything is quantized, including space and time, so we cannot subdivide them infinitely.
manner; they were simply proposed for practical purposes, but they allowed for the accurate prediction of the true distribution of blackbody radiation at all wavelengths. This meant that Planck’s constant was more than a convenient mathematical fluke, but a clue to the fundamental nature of reality. The fact that Planck’s constant is so small explains why the notion of quantization of energy had not appeared previously; it shows that energy is quantized on such a small scale that the gradations between the permitted values are miniscule, so as to appear non-existent to any measuring apparatus. Energy appears as though it is continuous at the macroscopic level of the observer, but it is quantized at the microscopic level. This conclusion was so strange that most scientists at the time, including Planck, did not believe it had concrete, physical meaning.

This was the first time that quantization had solved a problem in physics, but it would thereafter be used as the foundation of quantum physics. It was the first in a series of developments that would transform physics and, by extension, our perception of reality (Polkinghorne 1984). While Planck’s work solved one problem, it created another: why is there this quantization of energy? This marked the beginning of the quantum revolution.

9. Planck’s Formulation

In 1900, Planck presented a paper to the Deutsche Physikalische Gesellschaft in Berlin that helped to change the intrinsic character of physical science.28 His concern in this paper was with the mechanical, thermodynamic and electrodynamic processes associated with ‘ideal’ resonators inside an ‘ideal’ blackbody enclosure. This work, in a sense, represented an extension of the endeavours of many other classical physicists, such as Kirchhoff, Clausius, Maxwell, Stefan, Wien and Boltzmann, but it also represented personal audacity, introducing as it did the concept of discrete action that was later to develop into quantum physics (Kangro 1976). Though originally heuristic and theoretical, it was later demonstrated that Planck’s concept of discrete action is fundamental to an intrinsic understanding of the physical universe.

There are a number of ways one can approach Planck’s achievement; however, to appreciate the fundamental importance of his work it is instructive to follow Planck’s own formulation of his blackbody spectral law (see Klein 1961).

In March 1895, Planck presented a paper in which he discussed the problem of resonant scattering by an oscillating dipole (of dimension compatible with the wavelength), of a plane electromagnetic wave.29 What this scattering process offered Planck was a way of relating the equilibrium state radiation in a blackbody enclosure to the states of hypothetically ideal resonators, which Planck introduced as comprising the walls of an ideal blackbody enclosure.30 Planck studied the mechanisms of emission/absorption and of damping of these hypothetical resonators. He realised that, by disregarding mechanical damping and focusing on radiation damping, and by taking the mechanism for the system’s irreversibility as being the conversion of the incident plane waves into spherical waves, he could begin to understand the dynamics of the radiation equilibrium in the enclosure (Kuhn 1978).

Boltzmann, though, brought up a fundamental flaw in this approach, which forced Planck to change his attitude and become favourably disposed towards Boltzmann’s statistical mechanics. The equations of

29 Planck, M. (1896) Ann. d. Phys. (3), 57, 1. The intensity of the plane polarized light is independent of the orientation of the plane of polarization as well as the position and direction of the light. Inside the enclosure, the radiation is isotropic and homogeneous.
30 The state of thermodynamic equilibrium is that state in which the entropy of the system has the maximum value compatible with the total energy as fixed by the initial conditions of the system. Planck used the analogy of discrete ‘ideal’ resonators as constituting the enclosure’s walls because Kirchhoff had proven, in 1859, that the energy spectrum of a large blackbody enclosure is independent of the constituents, shapes and sizes of the materials making up its walls.
electrodynamics do not lead to a monotonic convergence to a state of thermal equilibrium. Nothing in the laws of electrodynamics prevents the inverse of the scattering processes from occurring. That is, there is implicit in Maxwell’s equations a possibility of reversibility. This important fact forced Planck to admit that statistical descriptions were necessary. It also compelled him to consider energy exchange by discrete means as opposed to continuous energy exchange.

Moreover, my suggestion that the oscillator was capable of exerting a unilateral, in other words irreversible, effect on the energy of the surrounding field, drew a vigorous protest from Boltzmann, who, with his wider experience in this domain, demonstrated that according to the laws of classical dynamics, each of the processes I considered could also take place in the opposite direction; and indeed in such a manner, that a spherical wave emitted by an oscillator could reverse its direction of motion, contract progressively until it reached the oscillator and be re-absorbed by the latter, so that the oscillator could then again emit the previously absorbed energy in the same direction from which the energy had been received.31

Planck was attracted to the blackbody radiation problem by the universal character of its spectral distribution, which “represents something absolute, and since I had always regarded the search for the absolute as the loftiest goal of all scientific activity, I eagerly set to work” (Planck 1949: 34-35). Kirchhoff had shown that the nature of the radiation in thermal equilibrium in an enclosure (cavity), whose walls are kept at a fixed temperature, is completely independent of the properties of any material bodies, including the walls, which are in equilibrium with the radiation. Several properties of the universal function of temperature and frequency, which describes this equilibrium spectral distribution, had already been established. In order to formulate these, it is convenient to introduce the function as:

$$\varrho(\nu, T)$$

where $\varrho(\nu, T) d\nu$ is the energy per unit volume in thermal radiation, at absolute temperature $T$, which lies in the frequency interval from $\nu$ to $\nu + d\nu$.

Experiments by Stefan in 1879, and theoretical work by Boltzmann in 1884, had established that

$$\int_0^\infty \varrho(\nu, T) d\nu = \sigma T^4$$

Here, $\varrho$ is the energy density, $\nu$ is frequency, $T$ is temperature and $\sigma$ is the Stefan-Boltzmann constant.

In 1893, Wien proved rigorously that the distribution function $\varrho(\nu, T)$ must be of the form

$$\varrho(\nu, T) = \nu^3 f(\nu/T)$$

In 1896, Wien, using a somewhat questionable theoretical argument, proposed that $f(\nu/T)$ was of the form

$$f(\nu/T) = A e^{-B\nu/T}$$

where $A$ and $B$ represent constants, giving

This is Wien’s spectral distribution law. This was found to be a reasonable representation of the blackbody spectral distribution known up to 1899. In 1899, Planck, continuing his work begun in 1895 on the analogy of the ‘ideal’ resonators, proved that the distribution function must be of the form

\[ q(\nu, T) = A \nu^3 e^{-B\nu/T} \]

where \( q(\nu, T) \) represents the average energy of the ‘ideal’ resonators at frequency \( \nu \), at cavity temperature. Though this equation applies to a hypothetical cavity wall composed of ‘ideal’ resonators, because of Kirchhoff’s 1859 proof regarding the constituents of the walls of a blackbody enclosure, once a valid frequency-temperature relation is found for the average resonator energy \( E(\nu, T) \), it can be applied to a cavity formed by any real material. Owing to his belief in the universality of the second law of thermodynamics, Planck attempted to discern the form of \( E(\nu, T) \) by examining the fundamental relationship between the energy and entropy of a system in a state of thermal equilibrium. He began with

\[ \frac{dS}{dE} = \frac{1}{T} \]

and, by differentiating again, he derived the equally important equation

\[ \frac{d^2S}{dE^2} = -\frac{1}{T^2} \left( \frac{dT}{dE} \right) \]

The derivative \( dE/dT \) could be evaluated by combining Wien’s spectral law with Planck’s distribution function, so that

\[ E(\nu, T) = \frac{A\nu^3}{8\pi} e^{-B\nu/T} \]

and

\[ \frac{dE}{dT} = \left( \frac{B\nu}{T^2} \right) E \]

The entropy – energy derivative then becomes

\[ \frac{d^2S}{dE^2} = -\frac{1}{B\nu E} \]

The simplicity of this entropy-energy derivative greatly impressed Planck, but spectral measurements taken (at the same time that Planck did his work in 1899) at high temperatures and low frequencies made it clear.

that Wien’s spectral law had serious empirical limitations. In 1900, working with new data, Planck derived a simple dependence, which agreed with the high temperature data, i.e., \( q(v, T) \propto T \), \( E(v, T) \propto T \), \( dE/dT \) is constant. Planck realised that these two cases were limiting cases, one for relatively high energy and the other for low energy. Subsequently, he formulated a single equation that sufficed to express both limiting cases by fitting the two cases together by means of a trial function, namely

\[
\frac{d^2S}{dE^2} = -(E(A + E))^{-1}
\]

Integration of this equation yields

\[
\frac{dS}{dE} = \left(\frac{1}{A}\right) \ln \left(\frac{A + E}{E}\right) + d
\]

where \( d \) is the constant of integration. Replacing \( dS/dE \) by \( 1/T \), the equation becomes

\[
\frac{1}{T} = \left(\frac{1}{A}\right) \ln \left(\frac{A + E}{E}\right) + d
\]

Evaluation of the constant of integration (i.e., looking at the equation as \( 1/T \) approaches 0) gives \( d = 0 \). Solving the last equation yielded an appropriate equation for the resonant energy, namely

\[
E(v, T) = \frac{A}{\left(e^{\lambda/T} - 1\right)}
\]

giving the following expression for the distribution function:

\[
q(v, T) = \frac{8\pi}{c^3} \frac{Av^3}{\left(e^{\lambda/T} - 1\right)}
\]

Returning to the limiting case encompassed in Wien’s spectral distribution law, Planck showed that \( A \), a (supposed) constant, was actually a function depending on \( v \), so that \( A = F(v) \), with \( F \) representing a (real) constant. Thus, \( f(v, T) \) takes the form

\[
f(v, T) = \frac{1}{\left(e^{\lambda v/T} - 1\right)}
\]

and \( q(v, T) \) takes the form

\[
q(v, T) = \frac{Gv^3}{\left(e^{\lambda v/T} - 1\right)}
\]
which can be evaluated for \( B \) and thereafter \( G \), giving

\[
B = \frac{h}{k}
\]

\[
G = \frac{8\pi v^2 h}{c^4}
\]

where \( k \) is Boltzmann’s constant (\( k = 1.38 \times 10^{-23} \text{ J/K} \)), and \( h \) is a new universal constant, now known as Planck’s constant (\( h = 6.626 \times 10^{-34} \text{ Js} \)).

Planck, as a theorist, was not satisfied by simply fitting a formulation to empirical data by “lucky intuition” (Planck 1949: 41) and began to put his concepts into a rigorous form. As mentioned previously, criticism by Boltzmann persuaded Planck to consider a statistical interpretation of entropy, and this approach yielded dramatic results.

Planck began by considering the \( N \) ‘ideal’ resonators comprising the walls of a hypothetical blackbody enclosure. Each resonator has an average energy \( \langle E \rangle \) and an average entropy \( \langle S \rangle \), giving total energy \( E_t \) of \( N(E) \) and total entropy \( S_t \) of \( N(S) \). It was here that Planck introduced Boltzmann’s famous expression for probability:

\[
S_t = k \ln W
\]

and followed the method Boltzmann devised previously to deal with the entropy divided among many oscillators.\(^{33}\) By proposing that the total energy \( E_t \) is composed of \( n \) discrete units of energy, each of amount \( e \), he was able to equate \( N(E) \) to \( ne \) and open up the approach to the fundamental problem of \( W \): how to calculate the number of ways \( n \) discrete units of energy can be distributed among \( N \) resonators. More fundamentally, Planck realised that previous limitations in his reasoning, as well as in Boltzmann’s work, could be removed by considering the energy transfer to be occurring, not continuously, but by discrete means.

Planck then had a way of relating \( W \) to \( N \) and \( n \). Beginning with

\[
W = \frac{(N + n - 1)!}{n! (N - 1)!}
\]

and making the approximation (on the assumption that both \( N \) and \( n \) are large)

\[
W \approx \frac{(N + n)!}{n! N!}
\]

and by then using Stirling’s approximation

\[
\ln(Z!) \approx Z \ln Z - Z
\]

he was able to express \( k \ln W \) as

\(^{33}\) Boltzmann originally introduced this statistical description as a computational aid. Later, however, he became sufficiently satisfied to consider its application as being more general.
Therefore, the entropy of a single ‘ideal’ resonator is

$$S = \frac{S}{N} = k \left\{ \left( 1 + \frac{E}{e} \right) \ln \left( 1 + \frac{E}{e} \right) - \left( \frac{E}{e} \right) \ln \left( \frac{E}{e} \right) \right\}$$

Planck transformed this entropy relation into an energy relation by taking the derivative

$$\frac{dS}{dE} = k \left( \frac{e}{e} \right) \ln \left( \frac{e + E}{E} \right)$$

and by again equating this to $1/T$, so then

$$\frac{1}{T} = k \left( \frac{e}{e} \right) \ln \left( \frac{e + E}{E} \right)$$

Solving for $E(\nu, T)$, he ended up with his previously derived expression

$$E(\nu, T) = \frac{e}{\left( e^{\nu/kT} - 1 \right)}$$

By returning to Wien’s distribution function

$$q(\nu, T) = \nu^3 f(\nu/T)$$

and his own expression

$$q(\nu, T) = \frac{8\pi \nu^2}{c^3} E(\nu, T)$$

Planck reasoned that $e \propto \nu$ or that $e = h\nu$. Thus, he was able to show rigorously his distribution function

$$q(\nu, T) = \frac{8\pi \nu^2}{c^3} \frac{h\nu}{\left( e^{\nu/kT} - 1 \right)}$$

To Planck’s satisfaction, these new radiation formulae agreed with the spectral data of Kurlbaum and Rubens, who at the time were doing precise spectral measurements at high temperatures and low frequencies. At frequencies that are sufficiently low (where $h\nu \ll kT$), we can write

$$\frac{h\nu}{e^{\nu/kT}} \approx 1 + \frac{h\nu}{kT}$$
and the above equation reduces to the classic Rayleigh-Jeans formula. Moreover, both the Stefan-Boltzmann law and Wien’s displacement law can be shown to follow from the equation.

Disconcertingly for Planck, though, the formulae implied that energy transfer appeared to be occurring by discrete steps, as opposed to the classical notion of continuous transfer. Even though he was the formulator of this important discovery, Planck refused for many years to accept the consequences of discrete energy transfer. It was left for someone as bold and imaginative as Einstein – the fox who could “perform acts and entertain ideas that are centrifugal rather than centripetal” (Berlin 1953: 2) – to expand further on Planck’s original work in the realm of statistical physics.

10. Conclusion

On 14 December 1900, Planck communicated his derivation of the following expression for the spectral energy density of blackbody radiation, which he later called “an act of desperation” (see Giulini and Straumann 2000).

\[
\varphi(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\left( e^{h\nu/kT} - 1 \right)}
\]

This expression successfully reproduced the energy density of blackbody radiation contained within the differential frequency interval \( \nu + d\nu \). Blackbody radiation is an electromagnetic phenomenon, so the radiation intensity depends on the speed of light, \( c \). It is also a thermal phenomenon, so it depends on the thermal energy, \( kT \), where \( T \) is the object’s temperature and \( k \) is Boltzmann’s constant. Moreover, blackbody radiation is a quantum phenomenon, so it depends on Planck’s constant, \( h \).

The physical principles behind this formula remained obscure, however, until illuminated during the following 25 years by Planck himself, by Einstein and others. Satyendra Nath Bose (1894-1974), for example, composed all the principles behind Planck’s formula – absolute entropy, quantized energy, and particle-like light quanta or photons – into a coherent and fully quantum theory of blackbody radiation. In 1924, he derived the spectral energy density of blackbody radiation by counting the microstates of a gas of photons. Subsequently, Einstein applied Bose’s method of counting microstates to a gas of particles with non-zero rest mass, and later predicted the existence of a new phase of matter called the Bose-Einstein condensate (see Dunningham and Vedral 2011; Lemons 2013).

Following Planck’s revolutionary concept of the quantization of energy, the next milestone came in 1905 when Einstein, while working as a clerk in the Swiss patent office in Bern, published four seminal papers that revolutionised physics. These papers concerned: the photoelectric effect; Brownian motion; special relativity; and mass-energy equivalence. It was for the first of these papers that Einstein was awarded the Nobel Prize in physics in 1921. This paper made use of quantization, which cemented Planck’s concept as

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34 It is instructive to note how similar the distribution functions are in form:

\[
\eta_k = \{ \exp[(\alpha\mu - \beta\epsilon_k) / \tau] + \delta \}^{-1}
\]

where

\( \eta_k \): the most probable number of particles occupying the \( k \)th orbital energy \( \epsilon_k \)

\( \tau \): the fundamental temperature

\( \mu \): chemical potential

\[
\begin{align*}
\{ \alpha = -1, \delta = -1 \} & \quad \text{Bose – Einstein} \\
\{ \beta = -1, \delta = 1 \} & \quad \text{Fermi-Dirac} \\
\{ \alpha = 0 \} & \quad \text{"corrected" Boltzmann}
\end{align*}
\]
more than just a fluke.

The photoelectric effect – which was first detected by Heinrich Hertz (1857-1894) in 1887 - was the observation that, if a metal plate is irradiated with ultraviolet radiation, an electron is ejected from the metal, which can be detected when it interacts with a positively charged wire or plate sensor. The incongruity was that the ability of the light to eject an electron depends only on its frequency, and not its intensity. So, if a beam of light was below a certain frequency, even a very bright beam could not eject an electron, whereas above a certain threshold frequency, the faintest beam could eject an electron. This seemed strange, but Einstein solved the problem by extending the concept of quantization of energy, developed by Planck, and rationalised that light must also be made of quanta, which he called photons. This explains the photoelectric effect, because electrons are ejected from a metal when they are struck by a single photon with sufficient energy. Even the faintest beam, if it is above a certain threshold frequency (and, hence, energy), will be able to do eject an electron from the metal. If none of the photons are of this minimum frequency (energy), then no matter how many photons strike the metal, an electron will not be ejected. It was found that the energy $E$ of the photon is given by

$$E = h\nu$$

where $h$ is Planck’s constant ($6.63\times10^{-34}$Js) and $\nu$ is the frequency of the photon. Thereafter, physicists had to accept the wave-particle duality nature of light, meaning that light behaves simultaneously as both a particle and a wave.

The photoelectric effect implied that is was not just the vibrational energy of the atoms in a blackbody that is quantized; light is also quantized, since photons are quanta. Moreover, since it had already been established that certain light-related phenomena, such as diffraction and interference, are best explained using a wave model, it must be the case that light can be described as both a particle and wave. This was the notion of wave-particle duality.

The subsequent milestone was reached by Niels Bohr (1885-1962) in 1913 when he showed that quantization of energy also applies to the energy of the electron in the hydrogen atom. He had gained inspiration from the insightful result obtained in 1885 by Johann Balmer (1825-1898), a Swiss schoolteacher, for the lines appearing in the spectrum of hydrogen. Bohr proposed that the electron can only inhabit specific energy levels, and that it will move between these energy levels when it absorbs or emits a photon of an energy that is equivalent to the difference in energy between the two energy levels involved in the transition. This model was able to explain the emission spectra of hydrogen and other elements, and by extension the colour of every object that reflects light.

Later, Louis de Broglie (1892-1987) demonstrated that it is not just light that exhibits wave-particle duality, but particles of matter as well. This meant that the electron, just like any other particle, has a wavelength that depends on its momentum, which complicated matters for chemistry. This notion was soon corroborated when a beam of electrons was shown to exhibit a diffraction pattern, just like a beam of light does. This denoted, therefore, that waves can act like particles, and particles can act like waves.

By all accounts, Planck was a talented cellist and pianist, and he might have imagined the quanta – discrete packets of energy – in the same way that a fixed number of harmonics is available to the vibrating string of a musical instrument. The resulting equation was simple, and it fit the experimental data. Introducing quanta of energy reduced the number of states of energy available to a system, and this solved the ultraviolet catastrophe. But Planck initially considered the quanta as a mathematical convenience – almost as a “trick” or “sleight of hand” – rather than something that was real. It was only when Einstein used the concept to explain the photoelectric effect in 1905, that Planck accepted that quanta were a physical property of light.

These new radiation formulæ aroused mixed feelings in Planck: they were mathematically pleasing to
him, since they were in agreement with the spectral data of Kurlbaum and Rubens, who conducted precise spectral measurements at high temperature and low frequencies; but they also engendered distress because, if one accepts the formulation, then energy transfer appeared to be occurring by discrete steps as opposed to the classical notion of continuous transfer.

Quantum physics (together with relativity) overthrew not only the old Newtonian physics but also undermined basic metaphysics (Kuhn 1978). Immanuel Kant (1724-1804) was perhaps the first philosopher to extend the notion of political and social revolution to the sciences. Kant taught that absolute Newtonian space and the principle of uniform causality are a priori principles of thought, necessary conditions on how human beings comprehend the world in which they live. Quantum physics, however, proved him totally mistaken. Cause and effect were mere appearance, and indeterminacy was at the root of reality.

As with several of the pioneers of quantum mechanics, Planck spent a significant amount of time struggling to come to terms with the consequences of his work, such as the idea that physical quantities might be discrete and not continuous, and the indeterminism inherent in quantum physics. While he was never in any doubt about the profound significance of what he had done, he was a cautious and restrained revolutionary (Brush 2002). Planck remained convinced that determinism and strict causality were essential requirements for physical science, and he found that some of the consequences of his equations often gave descriptions of physical reality that conflict with our everyday experience of the world.

The explanation of the second universal constant of the radiation law was not so easy. Because it represents the product of energy and time, I described it as the elementary quantum of action. … Either the quantum of action was a fictional quantity, then the whole deduction of the radiation law was in the main illusory and represented nothing more than an empty non-significant play on formulae, or the derivation of the radiation law was based on a sound physical conception. In this case the quantum of action must play a fundamental role in physics, and here was something entirely new, never before heard of, which seemed called upon to basically revise all our physical thinking, built as this was, since the establishment of the infinitesimal calculus by Leibniz and Newton, upon the acceptance of the continuity of all causative connections. Experiment has decided for the second alternative.35

35 Planck (1920).
References


